DIFFERENTIATION — Answers

1 **a** $f(x) = x(x+2)^{-1}$

$f'(x) = 1 \times (x+2)^{-1} + x \times [-(x+2)^{-2}]$

$= (x+2)^{-2}[(x+2) - x]$

$= \dfrac{2}{(x+2)^2}$

b $f'(x) = \dfrac{1\times(x+2) - x\times 1}{(x+2)^2}$

$= \dfrac{2}{(x+2)^2}$

2 **a** $= \dfrac{4\times(1-3x) - 4x\times(-3)}{(1-3x)^2}$

$= \dfrac{4}{(1-3x)^2}$

b $= \dfrac{e^x\times(x-4) - e^x\times 1}{(x-4)^2}$

$= \dfrac{e^x(x-5)}{(x-4)^2}$

c $= \dfrac{1\times(2x+3) - (x+1)\times 2}{(2x+3)^2}$

$= \dfrac{1}{(2x+3)^2}$

d $= \dfrac{\frac{1}{x}\times 2x - \ln x\times 2}{(2x)^2}$

$= \dfrac{1-\ln x}{2x^2}$

e $= \dfrac{1\times(2-x^2) - x\times(-2x)}{(2-x^2)^2}$

$= \dfrac{2+x^2}{(2-x^2)^2}$

f $= \dfrac{\frac{1}{2}x^{-\frac{1}{2}}\times(3x+2) - \sqrt{x}\times 3}{(3x+2)^2}$

$= \dfrac{(3x+2)-6x}{2\sqrt{x}(3x+2)^2} = \dfrac{2-3x}{2\sqrt{x}(3x+2)^2}$

g $= \dfrac{2e^{2x}\times(1-e^{2x}) - e^{2x}\times(-2e^{2x})}{(1-e^{2x})^2}$

$= \dfrac{2e^{2x}}{(1-e^{2x})^2}$

h $= \dfrac{2\times\sqrt{x-3} - (2x+1)\times\frac{1}{2}(x-3)^{-\frac{1}{2}}}{x-3}$

$= \dfrac{4(x-3)-(2x+1)}{2(x-3)^{\frac{3}{2}}} = \dfrac{2x-13}{2(x-3)^{\frac{3}{2}}}$

3 **a** $= \dfrac{2x\times(x+4) - x^2\times 1}{(x+4)^2}$

$= \dfrac{x^2+8x}{(x+4)^2}$

b $= \dfrac{\frac{1}{2}(x-4)^{-\frac{1}{2}}\times 2x^2 - \sqrt{x-4}\times 4x}{(2x^2)^2}$

$= \dfrac{x-4(x-4)}{4x^3\sqrt{x-4}} = \dfrac{16-3x}{4x^3\sqrt{x-4}}$

c $= \dfrac{2e^x\times(1-3e^x) - (2e^x+1)\times(-3e^x)}{(1-3e^x)^2}$

$= \dfrac{5e^x}{(1-3e^x)^2}$

d $= \dfrac{-1\times(x^3+2) - (1-x)\times 3x^2}{(x^3+2)^2}$

$= \dfrac{2x^3-3x^2-2}{(x^3+2)^2}$

e $= \dfrac{\frac{3}{3x-1}\times(x+2) - \ln(3x-1)\times 1}{(x+2)^2}$

$= \dfrac{3}{(3x-1)(x+2)} - \dfrac{\ln(3x-1)}{(x+2)^2}$

f $y = \dfrac{\sqrt{x+1}}{\sqrt{x+3}}$

$\dfrac{dy}{dx} = \dfrac{\frac{1}{2}(x+1)^{-\frac{1}{2}}\times\sqrt{x+3} - \sqrt{x+1}\times\frac{1}{2}(x+3)^{-\frac{1}{2}}}{x+3}$

$= \dfrac{(x+3)-(x+1)}{2(x+1)^{\frac{1}{2}}(x+3)^{\frac{3}{2}}} = \dfrac{1}{(x+1)^{\frac{1}{2}}(x+3)^{\frac{3}{2}}}$

4 **a** $\frac{dy}{dx} = \frac{2x\times(3-x)-x^2\times(-1)}{(3-x)^2}$

$= \frac{6x-x^2}{(3-x)^2}$

SP: $\frac{x(6-x)}{(3-x)^2} = 0$

$x = 0, 6$

$\therefore$ (0, 0), (6, −12)

b $\frac{dy}{dx} = \frac{4e^{4x}\times(2x-1)-e^{4x}\times 2}{(2x-1)^2}$

$= \frac{2e^{4x}(4x-3)}{(2x-1)^2}$

SP: $\frac{2e^{4x}(4x-3)}{(2x-1)^2} = 0$

$x = \frac{3}{4}$

$\therefore$ $(\frac{3}{4}, 2e^3)$

c $\frac{dy}{dx} = \frac{1\times\sqrt{2x+1}-(x+5)\times\frac{1}{2}(2x+1)^{-\frac{1}{2}}\times 2}{2x+1}$

$= \frac{(2x+1)-(x+5)}{(2x+1)^{\frac{3}{2}}} = \frac{x-4}{(2x+1)^{\frac{3}{2}}}$

SP: $\frac{x-4}{(2x+1)^{\frac{3}{2}}} = 0$

$x = 4$

$\therefore$ (4, 3)

d $\frac{dy}{dx} = \frac{\frac{1}{x}\times 2x-\ln 3x\times 2}{(2x)^2}$

$= \frac{1-\ln 3x}{2x^2}$

SP: $\frac{1-\ln 3x}{2x^2} = 0$

$x = \frac{1}{3}e$

$\therefore$ $(\frac{1}{3}e, \frac{3}{2}e^{-1})$

e $y = \frac{(x+1)^2}{(x-2)^2}$

$\frac{dy}{dx} = \frac{2(x+1)\times(x-2)^2-(x+1)^2\times 2(x-2)}{(x-2)^4}$

$= \frac{2(x+1)[(x-2)-(x+1)]}{(x-2)^3} = \frac{-6(x+1)}{(x-2)^3}$

SP: $\frac{-6(x+1)}{(x-2)^3} = 0$

$x = -1$

$\therefore$ (−1, 0)

f $\frac{dy}{dx} = \frac{2x\times(x+2)-(x^2-3)\times 1}{(x+2)^2}$

$= \frac{x^2+4x+3}{(x+2)^2}$

SP: $\frac{(x+1)(x+3)}{(x+2)^2} = 0$

$x = -3, -1$

$\therefore$ (−3, −6), (−1, −2)

5 **a** $x = 2 \ \therefore y = 4$

$\frac{dy}{dx} = \frac{2\times(3-x)-2x\times(-1)}{(3-x)^2}$

$= \frac{6}{(3-x)^2}$

grad = 6

$\therefore\ y - 4 = 6(x - 2)$

[$y = 6x - 8$]

b $x = 0 \ \therefore y = 2$

$\frac{dy}{dx} = \frac{e^x\times(e^x+1)-(e^x+3)\times e^x}{(e^x+1)^2}$

$= \frac{-2e^x}{(e^x+1)^2}$

grad = $-\frac{1}{2}$

$\therefore\ y - 2 = -\frac{1}{2}(x - 0)$

[$y = 2 - \frac{1}{2}x$]

c $x = 4 \ \therefore y = 2$

$\frac{dy}{dx} = \frac{\frac{1}{2}x^{-\frac{1}{2}}\times(5-x)-\sqrt{x}\times(-1)}{(5-x)^2}$

$= \frac{(5-x)+2x}{2\sqrt{x}(5-x)^2} = \frac{5+x}{2\sqrt{x}(5-x)^2}$

grad = $\frac{9}{4}$

$\therefore\ y - 2 = \frac{9}{4}(x - 4)$

[$y = \frac{9}{4}x - 7$]

d $x = -1 \ \therefore y = \frac{1}{2}$

$\frac{dy}{dx} = \frac{3\times(x^2+1)-(3x+4)\times 2x}{(x^2+1)^2}$

$= \frac{3-8x-3x^2}{(x^2+1)^2}$

grad = 2

$\therefore\ y - \frac{1}{2} = 2(x + 1)$

[$y = 2x + \frac{5}{2}$]

6 **a** $x = 1 \ \therefore y = 0$

$$\frac{dy}{dx} = \frac{-1\times(3x+1)-(1-x)\times 3}{(3x+1)^2}$$

$$= \frac{-4}{(3x+1)^2}$$

grad $= -\frac{1}{4}$

$\therefore$ grad of normal $= 4$

$\therefore \ y - 0 = 4(x - 1)$

$4x - y - 4 = 0$

b $x = -2 \ \therefore y = -4$

$$\frac{dy}{dx} = \frac{4\times\sqrt{2-x}-4x\times\frac{1}{2}(2-x)^{-\frac{1}{2}}\times(-1)}{2-x}$$

$$= \frac{4(2-x)+2x}{(2-x)^{\frac{3}{2}}} = \frac{8-2x}{(2-x)^{\frac{3}{2}}}$$

grad $= \frac{3}{2}$

$\therefore$ grad of normal $= -\frac{2}{3}$

$\therefore \ y + 4 = -\frac{2}{3}(x + 2)$

$2x + 3y + 16 = 0$

c $x = 3 \ \therefore y = 0$

$$\frac{dy}{dx} = \frac{\frac{2}{2x-5}\times(3x-5)-\ln(2x-5)\times 3}{(3x-5)^2}$$

$$= \frac{2}{(2x-5)(3x-5)} - \frac{3\ln(2x-5)}{(3x-5)^2}$$

grad $= \frac{1}{2}$

$\therefore$ grad of normal $= -2$

$\therefore \ y - 0 = -2(x - 3)$

$2x + y - 6 = 0$

d $x = 2 \ \therefore y = \frac{1}{2}$

$$\frac{dy}{dx} = \frac{1\times(x^3-4)-x\times 3x^2}{(x^3-4)^2}$$

$$= \frac{-4-2x^3}{(x^3-4)^2}$$

grad $= -\frac{5}{4}$

$\therefore$ grad of normal $= \frac{4}{5}$

$\therefore \ y - \frac{1}{2} = \frac{4}{5}(x - 2)$

$8x - 10y - 11 = 0$

7 **a** $$\frac{dy}{dx} = \frac{x^{-\frac{1}{2}}\times(x-2)-(2\sqrt{x}-3)\times 1}{(x-2)^2}$$

$$= \frac{(x-2)-\sqrt{x}(2\sqrt{x}-3)}{\sqrt{x}(x-2)^2}$$

$$= \frac{-(x-3\sqrt{x}+2)}{\sqrt{x}(x-2)^2}$$

at A and B, $\frac{dy}{dx} = 0 \ \therefore x - 3\sqrt{x} + 2 = 0$

b $(\sqrt{x} - 1)(\sqrt{x} - 2) = 0$

$\sqrt{x} = 1, 2$

$x = 1, 4$

$\therefore \ A\,(1, 1), \ B\,(4, \frac{1}{2})$